1. Fergusan, Chapters 22, problem 1.

2. Fergusan, Chapters 22, problem 5.


4. For the multinomial problem with $X \sim M(1, \mathbf{p}(\boldsymbol{\theta}))$, show that

$$I(\theta) = \mathbf{p}(\theta)' \mathbf{P}^{-1} \mathbf{p}(\theta).$$

5. Consider the model $Y_i = \alpha + \beta x_i + \epsilon_i$ for $i = 1, 2, \ldots, n$, where the $x_i$ are known numbers not all equal, the $\epsilon_i$ are i.i.d. $N(0, \sigma^2)$, and the parameters $(\alpha, \beta, \sigma^2)$ are unknown.

(a) What is the likelihood ratio test of the hypothesis $H_0 : \alpha = \beta$? Give its exact distribution.

(b) What is the likelihood ratio test of $H_0$ when $\sigma^2$ is known? Give its exact distribution.

6. Let $X_{ij}$ be independent random variables with $X_{ij} \sim \mathcal{P}(\lambda_{ij})$ for $i = 1, \ldots, n$ and $j = 1, \ldots, k$.

(a) Find the likelihood ratio test of $H_0 : \lambda_{ij} = j \lambda$ for some $\lambda > 0$ and $i = 1, \ldots, n$ and $j = 1, \ldots, k$, against $H_1 - H_0$, where $H_1 : \lambda$ is independent of $i$ (i.e. $\lambda_{ij} = \lambda_j$ for some numbers $\lambda_j > 0$), $i = 1, \ldots, n$ and $j = 1, \ldots, k$.

(b) Describe the asymptotic distribution of the likelihood ratio test statistic.