1. Fergusan, Chapter 4, Problem 1.

2. Fergusan, Chapter 5, Problem 1.

3. Fergusan, Chapter 5, Problem 5.

4. Fergusan, Chapter 6, Problem 2.

5. Fergusan, Chapter 6, Problem 6.

6. Let $X_1, X_2, \ldots$ be independent exponential random variables with means $\beta_1, \beta_2, \ldots$ respectively, and let $Z_n = X_1 + \cdots + X_n$. Show that if $\max_{1 \leq j \leq n} \beta_j^2 / \sum_{j=1}^{n} \beta_j^2 \to 0$ as $n \to \infty$, then $(Z_n - E(Z_n))/\sqrt{\text{Var}(Z_n)} \xrightarrow{D} \mathcal{N}(0,1)$.

7. Let $X_1, X_2, \ldots$ be independent Poisson random variables with means $\lambda_1, \lambda_2, \ldots$ respectively, and let $Z_n = X_1 + \cdots + X_n$. Show that $(Z_n - E(Z_n))/\sqrt{\text{Var}(Z_n)} \xrightarrow{D} \mathcal{N}(0,1)$ if and only if $\sum_{j=1}^{n} \lambda_j \to \infty$.

8. Suppose $X_1, X_2, \ldots$ are i.i.d. $\mathcal{N}(\mu, \sigma^2)$, and let $S_n^2 = \sum_{i=1}^{n} (X_i - \bar{X}_n)^2/(n-1)$. Show that $\sqrt{n}(S_n^2 - \sigma^2) \xrightarrow{D} \mathcal{N}(0,2\sigma^4)$. Hint: Show that $\sqrt{n}(S_n^2 - \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \mu)^2) \xrightarrow{P} 0$. 