1. Ferguson, Chapter 14, Problem 1.
   Also (e). The logistic distribution with density \( f(x) = e^{-(x-\theta)/(1 + e^{-(x-\theta)})^2} \), where \( \theta \) is a fixed real parameter.

2. Ferguson, Chapter 15, Problem 4.

3. Let \( X_1, \ldots, X_n \) be a sample from the Pareto distribution with density \( f(x \mid \theta) = \theta/((x + \theta)^2) \) for \( x > 0 \). For fixed \( 0 < p < 1 \), let \( x_p(\theta) \) denote the \( p \)th quantile of the distribution, and let \( X_{(np)} \) denote the sample \( p \)th quantile.
   (a) What is the asymptotic distribution of \( X_{(np)} \) as \( n \to \infty \)?
   (b) Find a constant \( c(p) \) such that \( \hat{\theta}_n = c(p)X_{(np)} \) is a consistent, asymptotically unbiased estimate of \( \theta \). For what value of \( p \) is the asymptotic variance of \( \theta_n \) a minimum?
   (c) Find the asymptotic distribution of \( M_n = \max(X_1, \ldots, X_n) \).

4. Suppose that \( F(x) = \Phi(x - \mu) \) in Example 6 of Chapter 14, so that we are sampling from a normal \((\mu, 1)\) distribution.
   (a) Find the asymptotic distribution of \( M_n \).
   (b) Show that \( \hat{\mu}_n \) is a consistent estimate of \( \mu \). That is
      \[ \hat{\mu}_n = M_n - \sqrt{2\log n} \overset{P}{\to} \mu. \]
      What is its asymptotic efficiency relative to \( \bar{X}_n \)?
   (c) Let \( m_n = \min\{X_1, \ldots, X_n\} \). Show that
      \[ m_n + \sqrt{2\log n} \overset{P}{\to} \mu. \]
   (d) Let \( \tilde{\mu}_n = (m_n + M_n)/2 \). Show that \( \tilde{\mu}_n \) is a consistent estimate of \( \mu \). What is its asymptotic efficiency relative to \( \bar{X}_n \)?

5. Use R or Splus (or a language of your choice) to conduct a simulation of limiting extreme value distributions. In class and homework, we found constants \( a_n \) and \( b_n \) such that \( (M_n - a_n)/b_n \overset{L}{\to} G(x) \) for some \( G \). For a given sample size \( n \), generate a large number of samples (\( N \) samples) from a base distribution, and compute the sample maximum \( M_n \) for each one. Then plot the histogram or density estimate of the \( (M_n - a_n)/b_n \).
   You might take \( N = 1000 \) and try this for \( n = 50, 200, \ldots \). Use the following distributions.
   (a) Standard normal (See Problem 4, when \( \mu = 0 \)).
   (b) Logistic (See Problem 1 (e), when \( \theta = 0 \)).

   You can experiment with different choices of \( n \) and \( N \). The goal is to see how large a sample is needed for the asymptotics to take effect.