1. Fergusan, Chapter 15, Problem 3.

2. Fergusan, Chapter 17, Problem 2.

3. Fergusan, Chapter 17, Problem 3.

4. The extreme value distribution is related to logistic regression through the concept of utility maximization. Suppose you have two options to choose from, and you choose the first if \( X_1 + \theta_1 > X_2 + \theta_2 \), where \( X_1 \) and \( X_2 \) are independent random variables with distribution function \( e^{-e^{-x}} \), and \( \theta_1 \) and \( \theta_2 \) are known constants. The idea is that \( X_i + \theta_i \) is the utility for choice \( i \), so the decision maker chooses to maximize utility. Let \( Y = 1 \) for the first choice and \( Y = 2 \) for the second.

(a) Show that 
\[
P(Y = 1) = \frac{e^{\theta_1}}{e^{\theta_1} + e^{\theta_2}}.
\]

(b) This model generalizes. Let \( X_1, \ldots, X_k \) be independent with the extreme value distribution, and suppose \( \theta_1, \ldots, \theta_k \) are known constants. Take \( Y = i \) if and only if \( X_i + \theta_i > X_j + \theta_j \) for all \( j \neq i \). Show that 
\[
P(Y = i) = \frac{e^{\theta_i}}{\sum_{j=1}^{k} e^{\theta_j}} \text{ for } i = 1, \ldots, k.
\]