Example 11.1.1 from Milton and Arnold.

Relative humidity (x) versus solvent evaporation (y)

Output from the R statistical software:

Coefficients:

| Estimate (Intercept) | Std. Error | t value | Pr(>|t|) |
|----------------------|------------|---------|----------|
| 13.63887             | 0.57898    | 23.56   | <2e-16   |
| -0.08006             | 0.01048    | -7.64   | 9.37e-08 |

Residual standard error: 0.8861 on 23 degrees of freedom
Multiple R-squared: 0.7173, Adjusted R-squared: 0.705
F-statistic: 58.36 on 1 and 23 DF, p-value: 9.372e-08

\[
S = \sqrt{\frac{\sum \chi^2}{n S_{xx}}}
\]
From the output of the R statistical software, we have:

1. Estimates:
   \[ \hat{\beta}_0 = 13.6389 \]
   \[ \hat{\beta}_1 = -0.0801 \]
   \[ S^2 = (0.8861)^2 \]

2. Fitted model:
   \[ \hat{y} = 13.6389 - 0.0801x \]
(3) Hypothesis test for $\beta_i$:

$H_0 : \beta_i = 0$

$H_1 : \beta_i \neq 0$

The $p$-value is $9.37 \times 10^{-8}$.

As the $p$-value is very small, we reject $H_0 : \beta_i = 0$.

Therefore, there is statistical evidence that solvent evaporation depends on relative humidity.
4. Hypothesis test for $\beta_0$:

$H_0: \beta_0 = 0$

$H_1: \beta_0 \neq 0$

The $p$-value is less than $2 \times 10^{-16}$.

As the $p$-value is very small, we reject $H_0: \beta_0 = 0$.

Therefore, there is statistical evidence that the intercept is different than zero.
5. 95% confidence interval for $\beta_1$:

$$\hat{\beta}_1 \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{S_{xx}}}$$

$$= -0.08006 \pm 2.069 \times 0.01048$$

$$= (-0.1017, -0.0584)$$

6. 95% confidence interval for $\beta_0$:

$$\hat{\beta}_0 \pm t_{\frac{\alpha}{2}} S \sqrt{\frac{\sum x^2}{n S_{xx}}}$$

$$= 13.63887 \pm 2.069 \times 0.57898$$

$$= (12.4410, 14.8368)$$