1. In a Bayesian analysis, we can reduce the effect of an outlier by replacing the normal distribution with a distribution with thicker tails. In this context, an outlying observation has the interpretation of arising from a long-tailed distribution rather than from a normal distribution with a large variance. By varying the degrees of freedom, one can also examine the sensitivity of the analysis to the normality assumption.

(a) Suppose that the data $Y$ below follow the $t$ distribution with mean $\mu$, scale $\sigma^2$, and 4 degrees of freedom.

$$49, -67, 8, 16, 6, 23, 28, 41, 14, 29, 56, 24, 75, 60, -48.$$ 

Under the noninformative prior $p(\mu, \sigma^2) \propto \sigma^{-2}$, use importance sampling to compute $E[\mu|Y]$, $E[\sigma^2|Y]$, and $P(\mu > 0|Y)$.

(b) Repeat (a) for $t$ distributions with 8, 16, 32, and $\infty$ degrees of freedom. Are the results robust to the number of degrees of freedom?