Homework 1
STAT 8320
Due February 7, 2007

Problem 1: Suppose that $y \sim N_4(\mu, \Sigma)$ where

$$
\mu = \begin{pmatrix} 1 \\ -3 \\ 5 \\ -8 \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} 4 & 0 & 0 & -6 \\ 0 & 16 & 8 & 0 \\ 0 & 8 & 9 & 0 \\ -6 & 0 & 0 & 25 \end{pmatrix}.
$$

a. Find the distribution of $y_3$.

b. Find the distribution of $2y_1 - 5y_2 + 3y_3$.

c. Find the joint distribution of $y_2$ and $3y_1 + y_2 - 2y_3$.

d. Find the conditional distribution of $y_4$ given $y_1 + y_4 = -4$.

e. Which of the following pairs are independent: i. $y_1$ and $y_2 - y_3$, ii. $(y_1, y_3)'$ and $(y_2, y_4)'$, iii. $y_1 + 3y_2$ and $y_3 + 4y_4$.

Problem 2: A study about the amount of vitamins in a cup of spinach measures the following variables:

$y_1$: Vitamin A (mg)

$y_2$: Vitamin K (mcg)

$y_3$: Calcium (mg)

$y_4$: Folate (mcg)

$y_5$: Potassium (mg)

Suppose that 30 samples are analyzed and give the following results:

$$
\bar{y} = \begin{pmatrix} 517.3 \\ 144.8 \\ 30.1 \\ 36.1 \\ 345.7 \end{pmatrix} \quad \text{and} \quad S = \begin{pmatrix} 27.0 & 4.5 & 3.8 & -1.4 & -1.5 \\ 4.5 & 12.3 & -0.4 & -0.9 & -0.8 \\ 3.8 & -0.4 & 5.2 & 1.3 & -1.4 \\ -1.4 & -0.9 & 1.3 & 5.3 & 0.0 \\ -1.5 & -0.8 & -1.4 & 0.0 & 15.3 \end{pmatrix}.
$$
a. Use the attached $d_i^2$ values to perform Mardia’s test for kurtosis.

b. Use the attached $d_i^2$ and $\chi^2$ values to draw a Q-Q plot. Do the data appear normal?

c. Test the null hypothesis $H_0: \mu = [515, 145, 30, 35, 345]'$.

d. Form simultaneous 95% Bonferroni confidence intervals for the five means.

e. A woman is planning to prepare a spinach salad for dinner. Assuming normality, that the woman uses exactly 1.5 cups of spinach for her salad and that she also acquires 200 mg of calcium from a multivitamin, find the mles of the mean and variance of the amount of calcium she received.

Problem 3: Many cholesterol medications are currently being advertised. Suppose that a study is undertaken to determine if there is a difference in the effectiveness of two such medications. 24 men are recruited to participate in the study, 12 will be assigned to each of the medications. Suppose that the following measurements are taken: initial cholesterol, cholesterol after 2 months, after 4 months, and after 6 months. For all parts, assume that the data are normally distributed and independent. The summary statistics for the two groups are below:

$$\bar{y}_1 = \begin{pmatrix} 259.8 \\ 170.3 \\ 146.4 \\ 147.3 \end{pmatrix}, \quad \bar{y}_2 = \begin{pmatrix} 245.4 \\ 162.1 \\ 160.9 \\ 164.6 \end{pmatrix}, \quad S_1 = \begin{pmatrix} 289.4 & 169.3 & 183.3 & 162.5 \\ 169.3 & 426.5 & 363.3 & 294.0 \\ 183.3 & 363.3 & 371.4 & 270.2 \\ 162.5 & 294.0 & 270.2 & 429.1 \end{pmatrix},$$

and

$$S_2 = \begin{pmatrix} 328.8 & 232.8 & 283.0 & 238.9 \\ 232.8 & 428.9 & 257.4 & 289.6 \\ 283.0 & 257.4 & 314.1 & 287.9 \\ 238.9 & 289.6 & 287.9 & 436.2 \end{pmatrix}$$

a. Test the null hypothesis that the two variance covariance matrices are equal.

b. Test the null hypothesis that there is no difference between the two means.

c. Test the null hypothesis that there is no difference between the initial cholesterol means for the two groups.

d. Test the null hypothesis that the profiles for the two medications are parallel for 2, 4, and 6 months.