Problem 1: Recall that the Poisson distribution has density

\[ f(y; \mu) = \frac{\mu^y \exp(-\mu)}{y!}. \]

Write this distribution in exponential form. Be sure to define the functions \( a(\cdot), b(\cdot), c(\cdot) \).

Find the mean and variance of this distribution using the exponential form.

What is the form of the canonical link for this distribution? Show that this form has the desired property of a canonical link.

Find the form of the scaled deviance, \( D^* \), for this distribution. Assume that your sample contains \( n \) independent observations.
Problem 2: A local nursery is investigating which factors influence the number of tomatoes seeds produced by a particular species of plant. It is believed the the following factors may influence the number:

\[ x_1 \] average daytime temperature,
\[ x_2 \] average overnight temperature,
\[ x_3 \] average monthly rainfall, and
\[ x_4 \] average hours of direct sunshine.

Thirty two plants are subjected to a variety of the above conditions and the number of tomatoes produced by each plant is recorded.

Look at the output provided. Which of the parameters are significantly different from zero?

Provide the point estimate and confidence interval for the coefficient of \[ x_2 \]. Interpret these values; make sure to mention whether or not your interval implies that overnight temperature significantly effects the number of tomatoes, and what the point estimate says about how changes in temperature effect the number of fruits.

Use the information provided to estimate the number of tomatoes produced by a plant with an average of 80 degrees in the day, 50 degrees overnight, 2 inches of precipitation and 9 hours of direct sunlight.
Problem 3: Consider the following model for the growth of a bacteria in a petri dish:

\[ Y_i = \theta_1^{x_{1i}} \exp(-\theta_2 x_{2i}) + \epsilon_i \]

Find the forms needed to perform one update of the Gauss-Newton algorithm. Fill in these forms for the three data values \((y, x_1, x_2) = (20, 3, 5), (12, 1, 7), (42, 5, 8)\). Note: you do not need do any simplification, but do write the new parameter estimates as a function of all of the above information and the current values of the \(\theta_s\).

Find the forms needed to perform one update of the Newton-Raphson algorithm. Fill in these forms for the three data values above.

Suppose that the maximum likelihood estimates are \(\hat{\theta}_1 = 1.35\) and \(\hat{\theta}_2 = .30\). Find an estimate of the error based upon the three data points above.
Problem 4: Suppose that a nonlinear model of the form

\[ Y_i = \frac{\theta_1 X_{1i} + \theta_2 X_{2i}}{\theta_3 X_{3i}} \]

is fit using some data. The attached output shows the results of this analysis.

Use the information provided to predict the response for an observation which has \( x_1 = .5, x_2 = 2, x_3 = 12. \)

Perform tests of the null hypotheses, \( H_{01} : \theta_1 = 0 \) and \( H_{02} : \theta_2 = 0. \) Why does it not make sense to test \( H_{03} : \theta_3 = 0? \)

Suppose that the constraint \( 0 < \theta_2 < 1 \) is fundamental to the model. Describe any steps that you would take to accommodate this constraint.