Problem 1: Suppose that $y_1$ is $N_p(\mu_1, \Sigma_1)$ and $y_2$ is $N_p(\mu_2, \Sigma_2)$. Find the distribution of $Cy_1 + Dy_2$, assuming that $C$ and $D$ are $m \times p$ matrices of constants and the two vectors are independent.

Problem 2: Assume that $A$, $B$, and $c$ are matrices or vectors of constants, and $x$ and $y$ are vectors of random variables. Show that $\text{var}(Ay + Bx + c) = A\Sigma_y A' + B\Sigma_x B' + A\Sigma_{yx} B' + B\Sigma_{yx} A'$. Hint: Use the definition of the variance-covariance matrix.
Problem 3: Suppose that you perform all three tests in the profile analysis. Provide an graphical example for two populations and three time points where

a. Parallelism is rejected, but neither coincidentalism or horizontalism are rejected.

b. Parallelism and horizontalism are not rejected, but coincidentalism is.

c. Parallelism and horizontalism are rejected, but coincidentalism is not.
Problem 4: Suppose that the SSCP for the test $H_{00} : C_0 \beta M = 0$, where

$$ C_0 = \begin{pmatrix} 1 & -2 & 1 \\ 1 & 0 & -1 \end{pmatrix}, \quad \beta = \begin{pmatrix} \mu_{11} & \mu_{12} & \mu_{13} & \mu_{14} \\ \mu_{21} & \mu_{22} & \mu_{23} & \mu_{24} \\ \mu_{31} & \mu_{32} & \mu_{33} & \mu_{34} \end{pmatrix}, \quad \text{and} \quad M = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} $$

is 2176.3. If we know that the SSCP for the test $H_{01} : C_1 \beta M = 0$, where $C_1 = (1 \ 0 \ -1)$, is 103.6, what can we say about

a. The SSCP for the test $H_{02} : C_2 \beta M = 0$, where $C_2 = (1 \ -2 \ 1)$?

b. The SSCP for the test $H_{03} : C_3 \beta M = 0$, where $C_3 = (1 \ -1 \ 0)$?

BONUS: The SSCP for the test $H_{04} : C_4 \beta M = 0$, where

$$ C_4 = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & -2 \end{pmatrix} $$
Problem 5: Look at the output for a factor analysis attached to this exam. In this analysis there are six total traits, cleverly labelled A, B, C, D, E, and F. It is believed that much of the variability observed in these traits could be explained by a small number of underlying factors. Towards this end, the attached analysis is considered.

a. Look at the SAS output. What are the estimates for the factor loadings for factor 1?

b. Suppose that the values of the first and second factor for an unknown observation are $-0.33$ and $2.86$. Assuming that the model is true, describe the relationship between these values and the values of the original traits.

c. Use the output to estimate the values of the six $\psi$s based upon analysis of the correlation matrix.

d. How would you interpret these values?