1. We wish to test the hypothesis $H_0 : \mu_1 \geq \mu_2$, versus $H_a : \mu_1 < \mu_2$, where $\mu_1$ represents the average price of homes with fewer than 2 bathrooms and $\mu_2$ is the true mean price for homes with at least 2 bathrooms. In order to create a grouping variable with only two levels, I used the SAS code:

```sas
data homes;
  input Obsvn Price Area Acres Rooms Baths;
  if baths eq 2.0 then group=1;
  if baths eq 1.0 then group=2;
datalines;
.
.
data homes2; set homes;
  if group ne .;
proc ttest data=homes2;
  class group;
  var price;
run;
```

A portion of the SAS output is shown below:

The TTEST Procedure

| T-Tests          | Method     | Variances | DF | t Value | Pr > |t| |
|------------------|------------|-----------|----|---------|------|---|
| Price            | Pooled     | Equal     | 65 | 5.36    | <.0001|
| Price            | Satterthwaite | Unequal  | 41.9 | 5.00 | <.0001|

Equality of Variances

<table>
<thead>
<tr>
<th>Variable</th>
<th>Method</th>
<th>Num DF</th>
<th>Den DF</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>Folded F</td>
<td>28</td>
<td>37</td>
<td>3.03</td>
<td>0.0018</td>
</tr>
</tbody>
</table>

**Conclusion:** The assumption of equal variances is violated, so one should use the Satterthwaite p-value. The test statistic (5) yields a (one-sided) p-value that is less than $\alpha$, so we reject $H_o$ and conclude that homes with 2 bathrooms are worth more than homes with just one one bathroom.

(Note: SAS command `out=cdf('T', -5, 42);` gives exact p-value=.000005323)

2. Let $X_1, \ldots, X_n$ be $\sim$ iid Normal($\mu, 9$). In class we saw that a size $\alpha = .01$ test of $H_o : \mu = 8$ versus $H_a : \mu \neq 8$ rejects $H_o$ if

$$\left| \frac{\sqrt{n}}{3} (\bar{X} - 8) \right| > z_{.005}.$$ 

Compute the Power function
\[ \Pi(\mu) = P(|\frac{\sqrt{n}}{3}(\bar{X} - 8)| > 2.5762) \leftarrow P(\text{Reject } H_0) \]
\[ = P(|\bar{X} - 8| > \frac{3}{\sqrt{n}} 2.5762) \]
\[ = P(|\bar{X} - 8| > 2.5762) \text{ since } n = 9 \]
\[ = P(\bar{X} > 8 > 2.5762) + P(\bar{X} < 8 < -2.5762) \]
\[ = P(\bar{X} > 10.5762) + P(\bar{X} < 5.4238) \]
\[ = P(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} > 10.5762 - \mu) + P(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < 5.4238 - \mu) \]
\[ = P(Z > 10.5762 - \mu) + P(Z < 5.4238 - \mu) \]
\[ = 1 - \Phi(10.5762 - \mu) + \Phi(5.4238 - \mu). \]

(1)

data power;
  input mu;
  power=1-cdf('normal',10.5763-mu)+cdf('normal',5.4238-mu);
datalines;
  3.00
  3.05
  3.10
  3.15
  ...
  12.90
  12.95
  13.00
;
proc gplot;
  plot power*mu;
  symbol1 interpol=join;
run;

Graph on last page.

3. Let \( \mu = \) true mean taste score for Cheddar cheese. We wish to test \( H_0 : \mu = 20 \) versus \( H_a : \mu > 20 \).

The TTEST Procedure

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Lower CL</th>
<th>Mean</th>
<th>Upper CL</th>
<th>Lower CL</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Std Dev</th>
<th>Std Dev</th>
</tr>
</thead>
</table>

T-Tests

| Variable | DF | t Value | Pr > |t| |
|----------|----|---------|------|---|
| taste2   | 29 | 1.53    | 0.1375 |

Since \( p\text{-value} = 0.1375 / 2 = 0.06875 \) is NOT less than \( \alpha = 0.05 \), there is insufficient statistical evidence to conclude that cheddar cheese has a higher mean average taste score.

4. This is a sample size question, in the setting of mean estimation. The appropriate formula is on page 40:

\[ n = \left( \frac{z_{0.025} \sigma}{E} \right)^2. \]

Plugging in \( \alpha = 0.05 \), \( z_{0.025} = 1.96 \), \( E = 1 \), \( \sigma = 7 \) (Range/4) we get \( n = 189 \).  

2
5. Let \( X_1, \ldots, X_n \) be iid Normal(\( \mu, \sigma^2 \)). It is desired to test \( H_0 : \sigma^2 \leq \sigma_o^2 \), versus \( H_a : \sigma^2 > \sigma_o^2 \).

We saw in class that the usual parametric test rejects \( H_0 \) when

\[
(n - 1)s^2/\sigma_o^2 > \chi^2_{a,n-1}.
\]

Derive an expression for the Power curve \( \Pi(\sigma) \) of this test.

\[
\Pi(\sigma) = P\left( \frac{(n - 1)s^2}{\sigma^2_o} > \chi^2_{a,n-1} \right) = P\left( \frac{(n - 1)s^2}{\sigma^2_o} > \frac{\sigma^2_o}{\sigma^2_o} \chi^2_{a,n-1} \right) = P(Y > \sigma^2_o \chi^2_{a,n-1})
\]

(2)

where \( Y \) is a Chi-Square random variable with \( n - 1 \) degrees of freedom.

6. For each of the experimental situations below, give the appropriate statistical method you would use, the test statistic, and the assumptions of the testing method.

(a) Two identical footballs, one air-filled and one helium-filled, were used outdoors on a windless day MU’s athletic complex. Thirty-nine kickers were not informed which football contained the helium. Each football was kicked by each kicker, with a coin being flipped to determine which ball came first. The measurement of interest was the distance of the kick. We seek to determine if the data show significant statistical evidence that helium footballs go further than air-filled ones.

This is the paired t-test. We wish to test \( H_0 : \mu_{D} = 0 \) versus \( H_a : \mu_{D} > 0 \), where \( \mu_{D} = \mu_{Helium} - \mu_{Air} \). Test statistic is \( TS = \frac{D}{s_D/\sqrt{n}} \), and the assumption is Normality.

(b) Each male singer in the NY Choral Society self-reported his height to the nearest inch. For these males, their voice parts are Tenor or Bass. The choral director conjectures that the mean height of bass singers is different from that of tenor singers.

This is the independent samples t-test. We wish to test \( H_0 : \mu_T = \mu_B \) vs. \( H_a : \mu_T \neq \mu_B \). Assumption is Normality. Formula for test statistic is either

\[
TS_1 = \frac{\bar{X}_T - \bar{X}_B}{\sqrt{\frac{s^2_T}{n_T} + \frac{s^2_B}{n_B}}} \quad \text{or} \quad TS_2 = \frac{\bar{X}_T - \bar{X}_B}{S_p \sqrt{\frac{1}{N_T} + \frac{1}{N_B}}},
\]

depending upon outcome of the F-test for equality of variances.

(c) A researcher conjectures that the presence of a floral scent can improve a person’s learning ability. In an experiment, 22 people worked through a set of two pencil-and-paper mazes of equal difficulty, once while wearing a floral-scented mask and once while wearing an unscented mask. Individuals were randomly assigned to wear the floral mask on either their first or second try. Participants put on their masks one minute before starting the first trial in each group to minimize any distracting effect. Testers then measured the length of time it took subjects to complete the two mazes.

This is the paired t-test. We wish to test \( H_0 : \mu_{D} = 0 \) versus \( H_a : \mu_{D} < 0 \), where \( \mu_{D} = \mu_{Scent} - \mu_{None} \). (since “better” means less time to completion.) Test statistic is \( TS = \frac{D}{s_D/\sqrt{n}} \), and the assumption is Normality.