3.3 The Composition Method for generating continuous r.v.’s

This approach is used when we can write the target distribution as a mixture of well known distributions, for instance a discrete mixture of two continuous r.v.’s.

So suppose that we have two p.d.f. \( f(x|\theta_1) \) and \( f(x|\theta_2) \), but we want to generate a value of the random variable \( X \) with (target) p.d.f. the mixture of the two, i.e.,

\[
f(x|\theta_1, \theta_2) = af(x|\theta_1) + (1-a)f(x|\theta_2),
\]

where \( 0 < a < 1 \), the mixing probability.

Now if \( X_i \sim f(x|\theta_i), i = 1, 2 \), then defining

\[
X = \begin{cases} 
X_1, & \text{with probability } a \\
X_2, & \text{with probability } 1 - a
\end{cases}
\]

we can easily see that \( X \) becomes a realization from the target distribution \( f(x|\theta_1, \theta_2) \).

Thus the algorithm for generating the mixture distribution becomes:

**Step 1:** Generate a value \( U \sim U(0,1) \).

**Step 2:** If \( U < a \), then generate \( X_1 \sim f(x|\theta_1) \) and set \( X = X_1 \), otherwise \( (U \geq a) \) generate \( X_2 \sim f(x|\theta_2) \) and set \( X = X_2 \).

**Note 3.8** Here’s a general definition for handling mixtures (Discrete mixture of continuous distributions).
Assume the target distribution is written as

\[
f(x|\theta_1, \ldots, \theta_k) = \sum_{i=1}^{k} p_i f(x|\theta_i),
\]

where \( p_i \), the probabilities for the mixture, which must be a valid p.m.f., i.e., \( \sum_{i=1}^{\infty} p_i = 1 \), and \( f(x|\theta_i) \) the \( i^{th} \) continuous distribution we are mixing. The p.m.f. for \( p_i \) can be Binomial, Poisson \( (k = +\infty) \), and so forth. Here’s the general algorithm:

**Step 1:** Generate a value \( U \sim U(0,1) \).

**Step 2:** If \( \sum_{j=1}^{i-1} p_j \leq U < \sum_{j=1}^{i} p_j, \) \( i = 1, 2, \ldots \), with \( p_0 = 0 \), then generate \( X_i \sim f(x|\theta_i) \) and set \( X = X_i \) as the realization for the target distribution.