Statistics 150
Ries
Exam #2
Friday, October 20, 2000
Fall 2000
100 points

Name: Solutions

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Instructions:
I. Show all work. Clearly indicate your final answer.
II. DO NOT write in the right of the margin.
III. Carry all computations to at least three decimal places.
IV. You may use the formula packet that you purchased from the University Bookstore. You
    may not have written anything (except your name) on this packet.
V. Each part of each question is worth 10 points.
VI. Any grade corrections or appeals must be made during or immediately following the
    class period in which the exam paper is returned. If you miss that class without prior
    approval you forfeit any right to grade appeals or corrections.

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Information about Poisson random variables:

\[ p(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \text{ for } x = 0, 1, 2, \ldots \quad \lambda = \mu = \sigma^2. \]

The table below gives \( P(X \leq k) \), where \( X \) is a Poisson random variable.
1. The number of minutes late that a city bus arrives at a drop off location is approximately uniformly distributed between zero and 15 minutes.

   (a) 95% of the time, a bus will be ____ minutes late or less.

\[ A = B \times 15 \]

\[ .95 = \left( x - 15 \right) \frac{1}{15} \]

\[ .95 = \frac{x}{15} \quad \Rightarrow \quad x = (.95)(15) = 14.25 \]

(b) Find the probability that a bus will be more than 12 minutes late.

\[ P(X > 12) = \left( 15 - 12 \right) \frac{1}{15} \]

\[ = \frac{3}{15} = .20 \]

2. Suppose that the number of earthquakes that occur in a certain region during one year approximately follows a Poisson distribution with a mean of 9.

   (a) What is the probability that this region will experience exactly 10 earthquakes?

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\[ \lambda = \mu = \sigma^2 = 9 \]

\[ \sigma = 3 \]

(b) If a random sample of 36 years is selected and the number of earthquakes that occurred in the region during each year observed, what is the probability that the sample will average at most 8.25?

\[ n = 36 \]

\[ P(X \leq 8.25) \]

\[ \frac{8.25 - 9}{3/\sqrt{36}} = -1.5 \]

\[ P(Z = -1.5) = P(Z \geq 1.5) \]

\[ = .5 - P(0 \leq z \leq 1.5) \]

\[ = .5 - .4332 = .0668 \]
3. A consumer advocate claims that 70% of cable television subscribers are not satisfied with their cable service and that 87% of cable television subscribers have considered switching to satellite television service.

(a) Suppose that 8 subscribers are selected at random. What is the probability that the sample will contain exactly 6 who have considered switching to satellite television service?

(b) Suppose that 25 cable subscribers are selected at random. What is the probability that the sample will contain more than 20 subscribers who are not satisfied with their cable service?

(c) Suppose that 100 cable subscribers are selected at random. What is the probability that the sample will contain at least 62 who are not satisfied with their cable service. (Use the normal distribution as an approximation and make the continuity correction.)

\[
\begin{align*}
\mu &= 0.70 \\
\sigma &= 4.5826 \\
\bar{x} &= \mu \pm 3\sigma \\
\ &= 70 \pm (3)(4.5826) \\
\ &= (56.25, 83.75) \\
\ &= (0.100) \\
\ &= 0.100 \\
\ &= 0.100 \\
\ &= 0.100 \\
\ &= 0.100 \\
\ &= 0.100
\end{align*}
\]
4. Suppose that, for the population of college students who were in a bar at 11:30 p.m. last Saturday night, their blood-alcohol levels at that time followed a bell-shaped (normal) frequency curve with a mean of .08 and a standard deviation of .02. Suppose that a person with a blood-alcohol level of .10 or higher is considered legally drunk.

(a) What percentage of these students were considered "legally drunk" at 11:30 p.m. last Saturday?

\[
\frac{P(X \geq 0.10)}{P(Z \geq 1)} = 0.5 - P(0 \leq Z \leq 1) = 0.5 - \Phi(1) = 0.3413
\]

(b) The least intoxicated 5% of students had blood-alcohol levels of

\[
Z = \frac{X - \mu}{\sigma}
\]

\[
Z = -1.645 \text{ from table}
\]

\[
X = 0.08 + (-1.645)(0.02) = 0.0471
\]

5. A random sample of 49 students spent an average of $250 on textbooks this semester with a standard deviation of $28. Find a 99% confidence interval for the average amount of money that all students spent on textbooks this semester.

\[
\bar{X} = $250
\]

\[
\bar{X} = 250 \pm Z_{0.005} \left(\frac{S}{\sqrt{n}}\right)
\]

\[
Z_{0.005} = 2.575
\]

\[
250 \pm 2.575 \left(\frac{28}{\sqrt{49}}\right) = 250 \pm 10.3
\]

\[
= (239.70, 260.30)
\]