Statistics 150, Fall 1998

Exam #3

Name: Key

Instructions:

I. Show all work. Clearly indicate your final answer.
II. DO NOT write in the right margin line.
III. Carry all computations to at least three decimal places.
IV. You may use the formula packet that you purchased from the University Bookstore. You may not have written anything (except your name) on this packet.
V. For hypothesis tests you must justify all conclusions by showing a picture of the rejection region (including the critical value from the appropriate table) or by determining the p-value.
VI. Point values are marked. The entire exam is worth 100 points.

1. In 1990, a chemical company produced an average of 880 pounds of a certain type of plastic per day. Last year, the company switched to a new, less expensive production process. In a random sample of 50 days from last year, the mean daily yield of plastic was 871 pounds with a standard deviation of 21 pounds. Do the data suggest that the mean daily yield of plastic was less last year than in 1990? Test at the .01 level of significance.

\[ H_0 : \mu = 880 \quad \text{vs.} \quad H_a : \mu < 880 \]

Supporting work:

\[ z = \frac{871 - 880}{21/\sqrt{50}} = -3.03 \]

\[ -2.326 < z < -3.03 \]

Conclusion (circle one): Accept \( H_0 \) \[ \checkmark \] Reject \( H_0 \)

Re-state your conclusion in words (without using any statistical terms):

There is significant evidence suggesting that last year's yield was less than in 1990.
2. In 1990, a random sample of 4,000 citizens yielded 3,000 who were in favor of gun control legislation. 
\[ n = 4000 \quad \bar{X} = 7500 \quad \hat{p} = .75 \quad \alpha = .01 \quad \frac{\alpha}{2} = .005 \]

(a) For the year 1990, find a 99% confidence interval for the proportion of all citizens who were in favor of gun control legislation.

\[
\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = .75 \pm 2.576 \sqrt{\frac{(0.75)(0.25)}{4000}}
\]

\[
= .75 \pm .0176 = (.7324, .7676)
\]

(b) Suppose we want to estimate the proportion of citizens today who are in favor of gun control legislation. How large a sample should we take in order to be 95% confident that our estimate will be within .01 of the true proportion?

\[
\frac{(1.96)^2(0.95)(0.05)}{(0.01)^2} = 720.3
\]

3. A professor claims that students at this university spend, on average, 5 hours per week studying. One of his students believes that the actual average amount of studying time is more than 5 hours per week. The student intends to conduct a test at the .05 level of significance by following a random sample of 36 students during a randomly selected week and measuring how many hours they spend studying. Assume that the population standard deviation of weekly study time is 2 hours. Determine the probability the student will commit a type II error when, in reality, the average amount of time students at this university spend studying is 5.5 hours.

\[ H_0: \mu = 5 \]
\[ H_A: \mu > 5 \]

\[ \bar{X}_0 = 5 + 1.645 \left( \frac{2}{\sqrt{36}} \right) = 5.5483 \]

\[ \beta = \frac{5.5483 - 5.5}{2/\sqrt{36}} = .1449 \]

\[ \beta = .5 + .0557 = .5557 \]
4. A test of $H_0: \mu = 20$ vs. $H_a: \mu \neq 20$ produced a test statistic of 2.75.
Compute or bound the p-value as closely as possible if ...

(a) the test statistic was based on a random sample of size 20; \(\text{small} \sqrt{\text{v}}\)

\[ \sqrt{v} = 19 \]

\[ 0.01 < p \text{-value} < 0.02 \]

use t-table with $\sqrt{\text{v}}$ degrees of freedom.

Multiply bounds by z.

(b) the test statistic was based on a random sample of size 200; \(\text{large} \sqrt{\text{v}}\)

\[ Z (\text{area above 2.75}) = Z (0.5 - 0.4770) = Z (0.023) = 0.006 \]

5. In a controlled laboratory experiment, separate random samples of 10 adults and 10 children were tested by a psychologist to determine the room temperature that each person finds most comfortable. The adults preferred an average temperature of 77.5 degrees with a sample variance of 4.5 while the children preferred an average temperature of 74.5 degrees with a sample variance of 2.5. Does this suggest that children prefer cold temperatures, \(\mu_c\), than adults? Test at the .05 level.

\[ H_0: \mu_a - \mu_c = 0 \quad \text{vs.} \quad H_a: \mu_a - \mu_c > 0 \]

Supporting work:

\[ t = \frac{(77.5 - 74.5) - 0}{\sqrt{\frac{1}{10} + \frac{1}{10}}} = 3.59 \]

\[ S^2_p = \frac{(9)(4.5) + (9)(2.5)}{10 + 10 - 2} = 3.5 \]

\[ t_{0.05, 18} = 1.739 \]

Conclusion (circle one): \( \text{Accept } H_0 \) \( \text{Reject } H_0 \)
6. Before instituting an employee complaint/suggestion program (in August 1992) in its manufacturing plant, a company randomly sampled seven workers and measured their productivity in terms of the number of items produced per day. A year after the start of the complaint program (August 1993), the productivity of these workers was re-evaluated. The following data was collected:

<table>
<thead>
<tr>
<th>Employee Number</th>
<th>August 1993</th>
<th>August 1992</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>00001</td>
<td>10</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>00002</td>
<td>9</td>
<td>11</td>
<td>2</td>
</tr>
<tr>
<td>00003</td>
<td>12</td>
<td>14</td>
<td>2</td>
</tr>
<tr>
<td>00004</td>
<td>8</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>00005</td>
<td>10</td>
<td>9</td>
<td>-1</td>
</tr>
<tr>
<td>00006</td>
<td>11</td>
<td>14</td>
<td>3</td>
</tr>
<tr>
<td>00007</td>
<td>11</td>
<td>13</td>
<td>2</td>
</tr>
<tr>
<td>Sum</td>
<td>71</td>
<td>79</td>
<td>10</td>
</tr>
<tr>
<td>Mean</td>
<td>10.143</td>
<td>11.286</td>
<td>1.429</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1.345</td>
<td>2.360</td>
<td>1.272</td>
</tr>
</tbody>
</table>

(The summary measures given above may provide more information than is necessary to work the problem.)

Do the data provide sufficient evidence that the complaint/suggestion program has been effective in increasing worker productivity? Test at the .10 level of significance.

\[ H_0: \mu_0 = 0 \quad \text{vs.} \quad H_A: \mu_0 > 0 \]

Supporting work:

\[ t = \frac{1.429 - 0}{1.272/\sqrt{7}} = 2.97 \]

\[ t_{1.6} = 1.440 \]

Conclusion (circle one): Accept \( H_0 \) \[ \text{Reject } H_0 \]

Re-state your conclusion in words (without using any statistical terms):

The data do provide sufficient evidence that the program increased productivity.