1. Consider the Winbugs Volume I: Seeds: Random effect logistic regression. Assume that observations \( r_i \) are independent Bin \((n_i, p_i)\). Assume that

\[ y_i = \alpha_0 + x_{i1}\alpha_1 + x_{i2}\alpha_2 + x_{i1}x_{i2}\alpha_{12} + b_i, \quad i = 1, 2, \ldots, n = 21, \]

where \( \epsilon_i \) are iid \( N(0, 1/\text{tau}) \) random errors, \((\alpha_0, \alpha_1, \alpha_2, \alpha_{12}, \tau)\) are unknown parameters. We use an independent prior for \((\alpha_0, \alpha_1, \alpha_2, \alpha_{12}, \tau)\):

\[ \alpha_j \sim N(0, \delta_j), \quad \text{for} \quad j = 0, 1, 2; \]
\[ \alpha_{12} \sim N(0, \delta_{12}), \]
\[ \tau \sim \text{Gamma}(a, b). \]

We use Winbugs to run MCMC simulation.

(a) Find the posterior means and variances of \((\beta_0, \beta_1, \beta_1, \beta_2, \sigma^2 = 1/\tau)\) if \( \delta_j = \delta_{12} = 0.0000001, a = b = 0.001 \). Plot the marginal posterior densities of \((\alpha_0, \alpha_1, \alpha_2, \alpha_{12}, \sigma^2)\).

(b) Find the posterior means and variances of \((\alpha_0, \alpha_1, \alpha_2, \alpha_{12}, \sigma^2 = \tau)\) if \( \delta_j = \delta_{12} = 0.001, a = b = 0.01 \).

(c) Do you think any of the logistic regression coefficients \( \delta_j \) or \( \delta_{12} \) is 0?


(a) Find Bayesian estimators of \( \alpha, \beta, \gamma \) and \( \tau \).

(b) Find Bayesian estimators of \( \alpha, \beta, \gamma \) and \( \tau \) if we change the presison parameters in the priors of \( \alpha, \beta, \gamma \) to 0.001 and \( \tau \sim \text{gamma}(0.01, 0.01) \).

(c) Do you think any of the parameters \( \alpha, \beta \) and \( \gamma \) might be 0?